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SUMMARY

Calculations are presented from an analytical investigation on the influence of ring stiffeners on the instability modes of orthotropic cylinders subject to compressive or bending loads. The analysis is performed by employing small-deflection theory and by modifying the equilibrium equation to include the effects of discrete ring stiffeners characterized by a bending stiffness that restrains radial deformation of the shell. These calculations indicate that the ring bending stiffness necessary to cause panel instability can be adequately determined by use of an analysis which does not include the discreteness of the rings. Comparison of the results of the calculations with an empirical ring-design criterion in common use indicates that the empirical formula can be either very conservative or very nonconservative depending on the cylinder geometry.

INTRODUCTION

A problem encountered in the design of axially stiffened, axially compressed cylinders is the determination of the size of the circumferential stiffening elements or rings required to prevent general instability failure of the cylinders. Common practice is to determine the size of rings by the empirical formula of reference 1, which gives the required ring bending stiffness to force cylinder failure to occur between rings. The formula of reference 1 is based on tests of small cylinders with relatively few stiffening elements. Such cylinders are not very representative of the cylinders used in contemporary aircraft and launch vehicles. The use of this formula in contemporary design therefore causes some concern.

An analytical study of the ring stiffness required to force cylinder failure to occur between rings is made in the present study. For this purpose an analysis incorporating a Donnell type theory similar to the one employed in references 2 and 3 was used. The mathematical treatment of the rings is approximate and the ring stiffnesses predicted by the calculations made herein cannot be considered conclusive. However, the calculations do indicate buckling characteristics which aid in the design of torsionally weak rings of axially

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stiffened cylinders and which give a ring design criterion that has a better theoretical basis than the one (ref. 1) now in general use.

SYMBOLS

A_{ij} elements of a determinant

a coefficient of deflection function

D_{Q_x}, D_{Q_y} transverse shear stiffnesses of cylinder wall in longitudinal and circumferential directions, respectively

D_x, D_y bending stiffness of cylinder wall in longitudinal and circumferential directions, respectively

D_{xy} twisting stiffness of cylinder wall

d stiffener spacing, $\frac{L}{n + 1}$

E_r Young's modulus of elasticity for ring stiffeners

E_x, E_y extensional stiffness of cylinder wall in longitudinal and circumferential directions, respectively

G_{xy} in-plane shear stiffness of cylinder wall

$H = \frac{\gamma \beta^4 D_y}{D_x}$

I_r moment of inertia of ring stiffeners

i,j,k,m,p integers

k_x critical-axial-compressive-stress coefficient, $\frac{N_x L^2}{\pi^2 D_x}$

L length of cylinder

$$L_D = \frac{D_x}{1 - \mu_x \mu_y} \frac{\partial^4}{\partial x^4} + \left(\frac{\mu_y D_x}{1 - \mu_x \mu_y} + 2D_{xy} + \frac{\mu_x D_y}{1 - \mu_x \mu_y}\right) \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{D_y}{1 - \mu_x \mu_y} \frac{\partial^4}{\partial y^4}$$

$$L_E = \frac{G_{xy}}{E_y} \frac{\partial^4}{\partial x^4} + \left(1 - \mu_x' \frac{G_{xy}}{E_x} - \mu_y' \frac{G_{xy}}{E_y}\right) \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{G_{xy}}{E_x} \frac{\partial^4}{\partial y^4}$$

L_E^{-1} inverse of L_E defined by $L_E^{-1}\left(L_E w\right) = L_E\left(L_E^{-1} w\right) = w$

$$M_p = \frac{p^4}{1 - \mu_x\mu_y} + \left[\frac{D_y\mu_x}{D_x\left(1 - \mu_x\mu_y\right)} + \frac{D_{xy}}{D_x}\right]2p^2\beta^2 + \frac{\beta^4 D_y}{D_x\left(1 - \mu_x\mu_y\right)}$$
$$+ \frac{p^4 Z^2}{\pi^4\left[p^4 + \left(\frac{E_y}{G_{xy}} - 2\mu_y'\right)p^2\beta^2 + \frac{E_y}{E_x}\beta^4\right]} + k_x p^2$$

N_x critical resultant normal force in axial direction

n number of rings

R radius of cylinder to midplane

s minor-determinant number whose value is one through number of rings, 1, 2, 3, 4, . . . n

w displacement in radial direction of middle surface of cylinder

x longitudinal coordinate of cylinder

y circumferential coordinate of cylinder

Z cylinder curvature parameter, $\frac{L^2}{R}\sqrt{\frac{E_y}{D_x}}$

β ratio of cylinder length to circumferential buckle length, L/λ

γ ratio of bending stiffness of ring to bending stiffness of cylinder wall in circumferential direction, $E_r I_r/D_y d$

γ_{cr} ratio of bending stiffness of ring to bending stiffness of cylinder wall in circumferential direction at which buckling in the panel and general instability modes coincide

$\delta(x - id)$ Dirac delta defined such that $\int_{-\infty}^{\infty} f(x)\,\delta(x - id)dx = f(id)$, where $\delta(x - id) = 0$ when $x \neq id$

δ_{ij} Kronecker delta, $\delta_{ij} = 0$ when $i \neq j$ and $\delta_{ij} = 1$ when $i = j$

λ circumferential buckle length

μ_x, μ_y Poisson's ratios associated with bending of cylinder wall

μ_x', μ_y' Poisson's ratios associated with extension of cylinder wall

RESULTS AND DISCUSSION

The calculations and results presented were made with the use of the stability criterion presented in appendix A. The criterion applies to orthotropic cylinders loaded in compression and is based on conventional small-deflection buckling theory (ref. 4); the rings are treated as discrete members located at the neutral axis of the cylinder; they are characterized by a single stiffness, a bending stiffness that restrains radial deformations of the shell (refs. 2 and 3). Calculations were made for cylinders covering a wide range of orthotropic stiffnesses in order to study the influence of the discreteness of rings on the instability modes of stiffened cylinders. Although the calculations were made only for cylinders loaded in compression, they will also apply with reasonable accuracy to cylinders subject to bending loads.

All results of the calculations made exhibited similar characteristics and, therefore, only a typical calculation is presented in tables I and II where theoretical buckling coefficients for a selected stiffened cylinder are given in terms of the natural parameters of the problem. The data of tables I and II are also given in figures 1 and 2 in the form of plots of the buckling coefficient against the ring stiffness parameter. Separate curves are given for different values of the curvature parameter and for different modes of buckling.

The solid curves of figure 1 denote buckling of the cylinder in the general instability mode and the short dashed lines denote buckling in the panel instability mode. It will be noted that the curves denoting general instability form an envelope curve which is independent of the discreteness of the rings. This curve can be calculated by conventional orthotropic analysis (refs. 5 and 6), which distributes the bending stiffness of the rings over the panel length between rings and treats it as an additional property of the cylinder wall. Note also that the effect of ring discreteness does not become important until the buckling coefficient for panel instability has been exceeded, except for the case of a cylinder with a single ring ($n = 1$) and even for this case the effect is negligible. Hence, cylinders can be adequately analyzed by theories which do not account for the discreteness of rings. Another interesting feature of figure 1 is that the general instability curve for a given cylinder levels off at a value of the buckling coefficient which is less than the buckling coefficient for panel instability of the cylinder with one additional ring. The buckling mode entails deflection of the discrete rings at low-values of the ring stiffness ratio but as the ring stiffness ratio increases the deflection of the rings approaches zero and buckling entails only deflection of the cylinder wall between rings.

Additional curves similar to those of figure 1 are given in figure 2, except that the individual curves denoting buckling by general instability are not included when they do not represent the governing mode of buckling. These curves represent the governing mode of buckling only for the cylinder with a single central ring. (See curves for $n = 1$.) The general features of the curves of figure 2 are the same as those of figure 1 of reference 2 which gives the buckling coefficient of ring-stiffened isotropic cylinders in torsion. In both investigations the buckling coefficient increases with increasing ring stiffness until the panel instability mode is reached, and the transition to the panel instability mode occurs by a change in mode, so that little or no interaction between the two modes of buckling is involved. This last feature is important because it indicates that cylinders can be adequately designed with an analysis such as the one of reference 6, which does not include the discreteness of the rings. Both instability modes of concern here are adequately discussed in reference 6, where for the panel instability calculation the cylinder is considered to have a length equal to the ring spacing and for the general instability calculation the properties of the ring are added to those of the wall in determining stiffness constants for the stability equation.

Other calculations similar to those presented in figures 1 and 2 and covering a wide range of cylinder wall stiffnesses were made in this investigation. The results of the calculations were similar to those presented for a particular set of wall geometries and substantiate the conclusions drawn therefrom.

A comparison of the results of the present theory with Shanley's criterion for ring stiffness (ref. 1) is given in figure 3 for the particular set of wall stiffnesses used in the calculations for figures 1 and 2. Figure 3 was constructed with the parameter $\left(\frac{d}{R}\right)^2 \frac{R}{\sqrt{D_x/E_y}}$ held constant; this procedure results in a constant value for the reinforcement ratio γ_{cr} when computed by the present analysis. Figure 3 indicates that Shanley's criterion may be either very conservative or very nonconservative compared with the present analysis depending upon the proportions of the cylinder. Additional comparisons similar to the one shown in figure 3 have been made for cylinders with other wall stiffnesses. Although the relative position of the two curves changes, the trend indicated in figure 3 was found to be typical. Shanley's criterion was found to be more nonconservative at the larger values of d/R which correspond to small values of $\frac{R}{\sqrt{D_x/E_y}}$. Such proportions result in a small number of circumferential waves when the cylinder buckles. Hence it is concluded that Shanley's criterion is nonconservative for cylinders which buckle into modes with a small number of circumferential buckles.

CONCLUDING REMARKS

Results of theoretical calculations investigating the influence of discrete ring stiffness on the instability of orthotropic cylinders in compression or bending have been presented and discussed. A ring-stiffened cylinder can be strengthened by increasing the ring stiffness up to a limit at which the cylinder fails between the rings. The limit or transition from general to panel instability is essentially independent of the discreteness effect of the rings and allows the cylinder to be adequately analyzed and designed by conventional orthotropic analysis which distributes the properties of the rings. A comparison of the analysis with an empirical ring criterion of Shanley's shows that Shanley's criterion becomes nonconservative for cylinders which buckle into modes with a small number of circumferential buckles.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 17, 1964.

APPENDIX A

THEORETICAL SOLUTION

The equation of equilibrium governing the buckling of a curved orthotropic plate with infinite transverse shear stiffnesses $\left(D_{Q_x} = D_{Q_y} = \infty\right)$ and subject to in-plane axial stresses only is (ref. 4)

$$L_D w + \frac{G_{xy}}{R^2} L_E^{-1} \frac{\partial^4 w}{\partial x^4} - N_x \frac{\partial^2 w}{\partial x^2} + \sum_{i=1}^{n} E_r I_r \frac{\partial^4 w}{\partial y^4} \delta(x - id) = 0 \tag{A1}$$

The fourth term of equation (A1) is added to represent the radial restoring force due to the bending stiffness of equally spaced rings. (See refs. 2 and 3.) Endowing each ring with a single bending stiffness is equivalent to considering the ring to be without torsional stiffness and attached to the shell by a frictionless bond which maintains contact but allows the shell to slide freely under the ring. The equation of equilibrium may be solved by the Galerkin method in a manner similar to that of references 7 and 8.

The simple support boundary conditions considered herein are satisfied if the solution is taken as the infinite series deflection function, that is,

$$w = \sin \frac{\pi y}{\lambda} \sum_{m=1}^{\infty} a_m \sin \frac{m\pi x}{L} \tag{A2}$$

where the coefficients a_m are to be determined. When equation (A2) is substituted into the equilibrium equation, the Galerkin method of solution yields the equation

$$M_p a_p + \frac{2d}{L} H \sum_{i=1}^{n} \sin \frac{p\pi i}{n + 1} \sum_{k=1}^{\infty} a_k \sin \frac{k\pi i}{n + 1} = 0 \tag{A3}$$

where

$p = 1, 2, 3, \ldots$

$$M_p = \frac{p^4}{1 - \mu_x\mu_y} + \left[\frac{\mu_x D_y}{D_x\left(1 - \mu_x\mu_y\right)} + \frac{D_{xy}}{D_x}\right] 2p^2\beta^2 + \frac{D_y\beta^4}{D_x\left(1 - \mu_x\mu_y\right)}$$

$$+ \frac{p^4 Z^2}{\pi^4\left[p^4 + \left(\frac{E_y}{G_{xy}} - 2\mu_y{}'\right)\beta^2 p^2 + \frac{E_y}{E_x}\beta^4\right]} + k_x p^2$$

d ring spacing

n number of rings

$$H = \frac{\gamma\beta^4 D_y}{D_x}$$

$$\beta = L/\lambda$$

$$k_x = \frac{N_x L^2}{\pi^2 D_x}$$

$$Z = \frac{L^2}{R}\sqrt{\frac{E_y}{D_x}}$$

$$\gamma = \frac{E_r I_r}{D_y d}$$

Employing the identity

$$\sum_{i=1}^{n} \sin\frac{p\pi i}{n + 1} \sin\frac{k\pi i}{n + 1} = \frac{n + 1}{2}\delta_{npk}$$

where, if $p - k$ is a multiple of $2(n + 1)$,

$$\delta_{npk} = +1$$

if $p + k$ is a multiple of $2(n + 1)$,

$$\delta_{npk} = -1$$

and if neither or both are true,

$$\delta_{npk} = 0$$

equation (A3) becomes

$$M_p a_p + H\sum_{k=1}^{\infty} a_k \delta_{npk} = 0 \tag{A4}$$

where $p = 1, 2, 3, \ldots$. Equation (A4) gives the criteria for buckling if the determinant of the coefficients vanishes, that is,

$$\left|A_{ij}\right| = 0 \tag{A5}$$

The determinant obtained by expanding equation (A4) can be factored into minor determinants which correspond to the buckling modes of general and panel instability. These expressions are:

(1) General instability for a finite number of rings:

$$\left|A_{ij}\right| = \left|M_p\delta_{ij} + (-1)^{(i+j)}H\right| = 0 \tag{A6}$$

where, when i is odd

$$p = (i - 1)(n + 1) + s$$

and, when i is even

$$p = i(n + 1) - s$$

and the minor determinant number s equals 1, 2, 3, . . . n.

(2) General instability for an infinite number of rings which corresponds to orthotropic theory where the ring stiffness is added to the stiffness of the cylinder wall:

$$M_p + H = 0 \tag{A7}$$

where $p = 1, 2, 3, \ldots$.

(3) Panel instability:

$$M_p = 0 \tag{A8}$$

where $p = j(n + 1)$ and $j = 1, 2, 3, \ldots$.

The calculations required to solve the instability expressions, equations (A6), (A7), and (A8), for the critical-stress coefficient k_x were made for a large range of orthotropic cylinder stiffnesses. For the calculations presented herein a typical set of cylinder stiffnesses was chosen. These values are given in the figures and tables at the back of the paper. In order to calculate the critical-stress coefficient associated with each instability mode, the stability expressions are minimized with respect to the parameter β, which is a function of the cylinder length and the number of circumferential waves q in the buckling mode. Theoretically q must be zero or an integer number larger than one. However, negligible errors are usually involved when the stability expressions are minimized with respect to β instead of with respect to q, unless q is a small number. The value of q was found to be sufficiently large so that negligible errors resulted in the calculations presented. The required numerical calculations and minimizations were quite lengthy, and, therefore, were performed on a high-speed digital computer. The exact critical-stress coefficient was not found, but increments of β were taken suitably small so that only slight errors resulted.

The results obtained by solving equation (A6) are presented in the $n = 1$ through six blocks and $n = 1$ through five blocks of tables I and II, respectively. A representative example of the minimization of equation (A6) for three rings is given in figure 4. Examination of figure 4 shows that for the curve for $s = 1$ there are two minimums resulting from a mode change. The curve for $s = 2$ has a single minimum, while the curve for $s = 3$ produces the absolute minimum and is, therefore, the value recorded in table I. The types of results shown in figure 4 did not occur for every case, but the range of β for the calculations was taken to cover a large enough interval so that the possibility of double minimums was checked.

The axial buckle shape associated with the minimums of figure 4 are of interest, and are given in figure 5. The equation for the buckle shape as well as the values of k_x and β associated with the buckle shape are given in the figure.

Most of the solutions for the buckling coefficients presented herein were made by use of 4th-order determinants. A 10th-order determinant was used in selected solutions to check the convergence of the calculations. The 10th-order determinant calculations did not change the values of k_x in tables I and II by more than 0.01 from that obtained by using the 4th-order determinant, so that the convergence obtained with a 4th-order determinant was considered good.

The results obtained by solving equations (A7) and (A8) are given in tables I and II under the column headings of $n = \infty$ and panel instability, respectively. Representative examples of the minimization of equations (A7) and (A8) are given in figures 6 and 7, respectively. The mode shapes involved are depicted in the upper right-hand corner of the figures. The results are typical of those found in many shell-buckling problems. Equations (A7) and (A8) are equivalent to the equation derived by Stein and Mayers in reference 5 if the appropriate wall stiffnesses are employed in the Stein and Mayers equation when general and panel instability modes of buckling are computed.

REFERENCES

1. Shanley, F. R.: Weight-Strength Analysis of Aircraft Structures. McGraw-Hill Book Co., Inc., 1952, pp. 65-71.

2. Stein, Manuel; Sanders, J. Lyell, Jr.; and Crate, Harold: Critical Stress of Ring-Stiffened Cylinders in Torsion. NACA Rep. 989, 1950. (Supersedes NACA TN 1981.)

3. Batdorf, S. B.; and Schildcrout, Murry: Critical Axial-Compressive Stress of a Curved Rectangular Panel With a Central Chordwise Stiffener. NACA TN 1661, 1948.

4. Stein, Manuel; and Mayers, J.: A Small-Deflection Theory for Curved Sandwich Plates. NACA Rep. 1008, 1951. (Supersedes NACA TN 2017.)

5. Stein, Manuel; and Mayers, J.: Compressive Buckling of Simply Supported Curved Plates and Cylinders of Sandwich Construction. NACA TN 2601, 1952.

6. Card, Michael F.: Bending Tests of Large-Diameter Stiffened Cylinders Susceptible to General Instability. NASA TN D-2200, 1964.

7. Batdorf, S. B.: A Simplified Method of Elastic-Stability Analysis for Thin Cylindrical Shells. NACA Rep. 874, 1947. (Formerly included in NACA TN's 1341 and 1342.)

8. Duncan, W. J.: The Principles of the Galerkin Method. R. & M. No. 1848, British A.R.C., 1938.

TABLE I.- BUCKLING PARAMETERS FOR RING-STIFFENED ORTHOTROPIC CYLINDERS

$\mu_x = \mu_y' = 0$
$D_y/D_x = 0.00132$
$D_{xy}/D_x = 0.0477$
$E_y/G_{xy} = 3.0$
$E_y/E_x = 0.376$
$Z = 452.8$

γ	n = 1			n = 2			n = 3		
	k_x	β	s	k_x	β	s	k_x	β	s
1	11.11	5.6	1	11.12	5.6	1	11.13	5.6	1
3	13.28	5.2	1	13.34	5.2	1	13.35	5.2	1
10	18.25	4.7	1	18.52	4.6	1	18.56	4.6	1
30	20.54	7.8	1	25.58	7.3	2	27.47	5.3	2
62.5	20.56	7.8	1	26.13	8.0	2	32.66	7.1	3
100	20.57	7.8	1	26.26	8.1	2	33.39	7.9	3
300	20.58	7.8	1	26.38	8.2	2	33.89	8.3	3
1000	20.58	7.8	1	26.42	8.2	2	34.04	8.4	3

γ	n = 4			n = 5			n = 6		
	k_x	β	s	k_x	β	s	k_x	β	s
1	11.13	5.6	1	11.13	5.6	1	11.13	5.6	1
3	13.35	5.2	1	13.35	5.2	1	13.35	5.2	1
10	18.58	4.6	1	18.58	4.6	1	18.58	4.6	1
30	27.51	4.0	1	27.52	4.0	1	27.53	4.0	1
62.5	34.29	4.8	2	34.40	4.8	2	34.44	4.8	2
100	39.14	5.2	3	39.59	5.1	3	39.72	5.1	3
300	42.97	8.1	4	51.58	4.7	4	52.25	4.6	4
1000	43.37	8.4	4	54.35	8.2	5	65.84	3.9	5

γ	n = ∞			Number of rings	Panel instability		
	k_x	β	p		k_x	β	p
1	11.13	5.6	1				
3	13.35	5.2	1	1	14.89	7.5	2
10	18.58	4.6	1	2	21.35	8.2	3
30	27.53	4.0	1	3	29.25	8.5	4
62.5	34.49	4.8	2	4	38.75	8.6	5
100	39.80	5.0	3	5	49.94	8.5	6
300	52.53	4.5	4	6	62.91	8.3	7
1000	66.82	3.7	5				

TABLE II.- BUCKLING PARAMETERS FOR RING-STIFFENED ORTHOTROPIC CYLINDERS

$$\mu_x = \mu_y' = 0$$
$$D_y/D_x = 0.00132$$
$$D_{xy}/D_x = 0.0477$$
$$E_y/G_{xy} = 3.0$$
$$E_y/E_x = 0.376$$

Z	γ	n = 1			n = 2			n = 3			n = 4		
		k_x	β	s	k_x	β	s	k_x	β	s	k_x	β	s
30	0.1	1.876	2.2	1	1.876	2.2	1	1.876	2.2	1	1.876	2.2	1
	1	1.901	2.1	1	1.901	2.1	1	1.901	2.1	1	1.901	2.1	1
	10	2.086	1.9	1	2.087	1.9	1	2.087	1.9	1	2.087	1.9	1
	10^2	2.807	1.4	1	2.811	1.4	1	2.811	1.4	1	2.811	1.4	1
	10^3	4.444	.9	1	4.459	.9	1	4.461	.9	1	4.461	.9	1
	10^4	6.982	.6	1	6.161	.4	2	6.161	.4	2	6.161	.4	2
10^2	.1	3.478	3.5	1	3.478	3.4	1	3.478	3.5	1	3.478	3.5	1
	1	3.651	3.4	1	3.652	3.4	1	3.652	3.4	1	3.652	3.4	1
	10	4.763	2.9	1	4.775	2.9	1	4.776	2.9	1	4.777	2.9	1
	10^2	8.598	2.2	1	8.772	2.1	1	8.785	2.1	1	8.788	2.1	1
	10^3	11.19	3.9	1	14.31	1.8	2	14.64	1.7	2	14.67	1.7	2
	10^4	11.21	3.9	1	16.97	4.0	2	18.97	.9	3	19.00	.9	3
10^3	.1	18.53	7.5	1	18.53	7.5	1	18.53	7.5	1	18.53	7.5	1
	1	21.76	7.1	1	21.79	7.0	1	21.80	7.0	1	21.80	7.0	1
	10	33.04	10.6	1	38.65	10.8	2	40.07	5.8	1	40.12	5.8	1
	10^2	33.05	10.6	1	39.40	11.4	2	47.77	11.9	3	57.85	12.4	4
10^4	.1	143.9	14.0	1	143.9	14.0	1	143.9	14.0	1	143.9	14.0	1
	1	183.4	13.1	1	183.8	13.1	1	183.9	13.1	1	183.9	13.1	1
	10	197.3	22.1	1	204.1	24.6	2	216.1	27.3	3	233.6	29.2	4

Z	γ	n = 5			n = ∞		
		k_x	β	s	k_x	β	p
30	0.1	1.876	2.2	1	1.876	2.2	1
	1	1.901	2.1	1	1.901	2.1	1
	10	2.087	1.9	1	2.087	1.9	1
	10^2	2.811	1.4	1	2.811	1.4	1
	10^3	4.461	.9	1	4.461	.9	1
	10^4	6.161	.4	2	6.161	.4	2
10^2	.1	3.478	3.5	1	3.478	3.5	1
	1	3.652	3.4	1	3.652	3.4	1
	10	4.777	2.9	1	4.777	2.9	1
	10^2	8.788	2.1	1	8.789	2.1	1
	10^3	14.67	1.7	2	14.68	1.7	2
	10^4	19.00	.9	3	19.00	.9	3
10^3	.1	18.53	7.5	1	18.53	7.5	1
	1	21.80	7.0	1	21.80	7.0	1
	10	40.14	5.8	1	40.16	5.8	1
	10^2	69.62	12.9	5	87.16	7.3	4
10^4	.1	143.9	14.0	1	143.9	14.0	1
	1	183.9	13.1	1	183.9	13.1	1
	10	253.8	30.0	5	397.5	10.6	1

Z	Number of rings	Panel instability		
		k_x	β	p
30	1	4.912	2.1	2
	2	9.790	1.7	3
	3	16.57	.6	4
	4	25.37	.1	5
10^2	1	6.962	4.0	2
	2	12.08	4.0	3
	3	18.99	3.7	4
	4	27.77	3.3	5
10^3	1	25.08	9.8	2
	2	33.13	11.1	3
	3	42.34	11.9	4
	4	53.12	12.4	5
	5	65.32	12.6	6
10^4	1	158.3	20.0	2
	2	177.9	23.0	3
	3	198.6	26.0	4
	4	220.6	29.0	5
	5	243.4	30.0	6

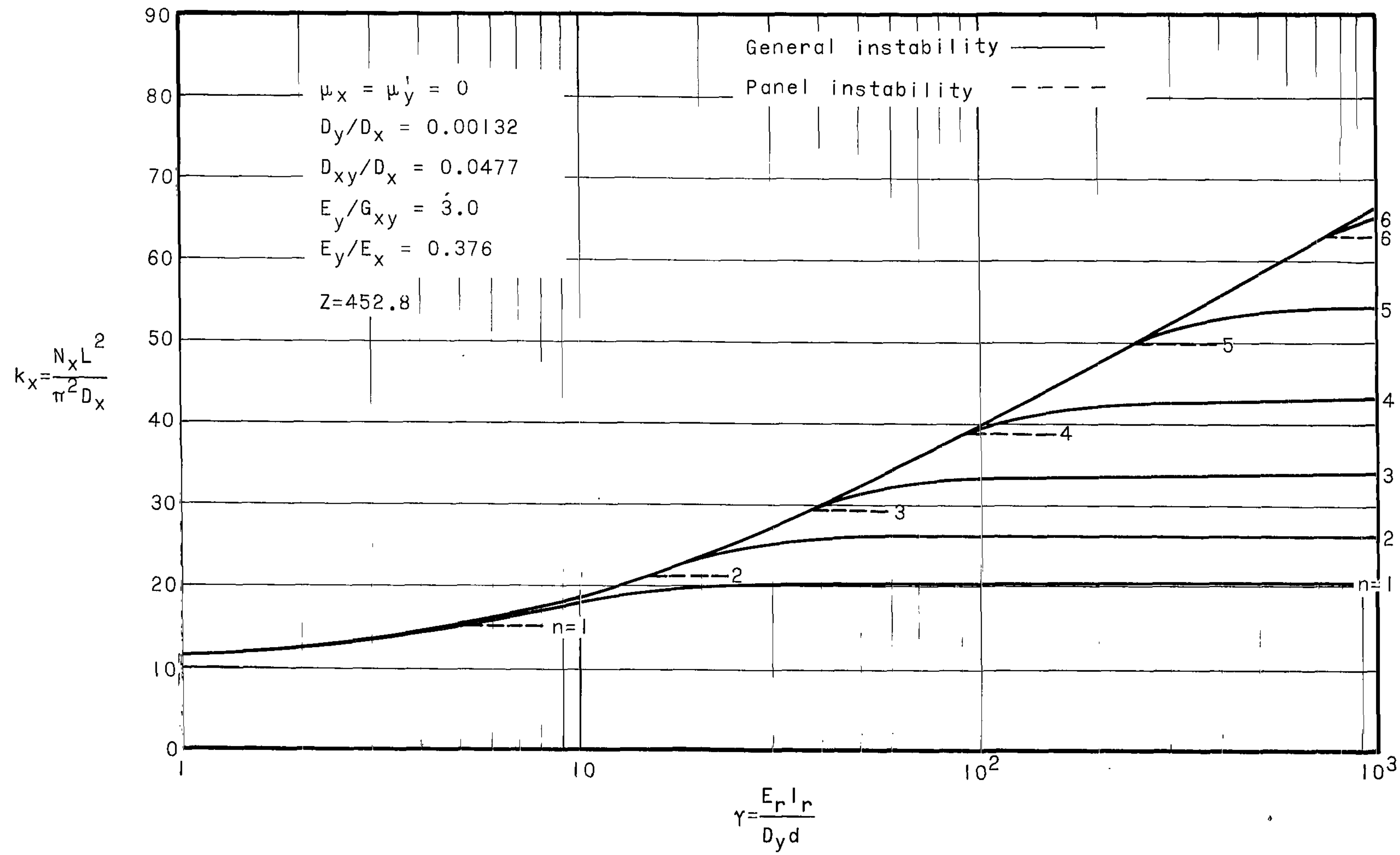


Figure 1.- Theoretical critical-stress coefficient of a ring-stiffened orthotropic cylinder in axial compression.

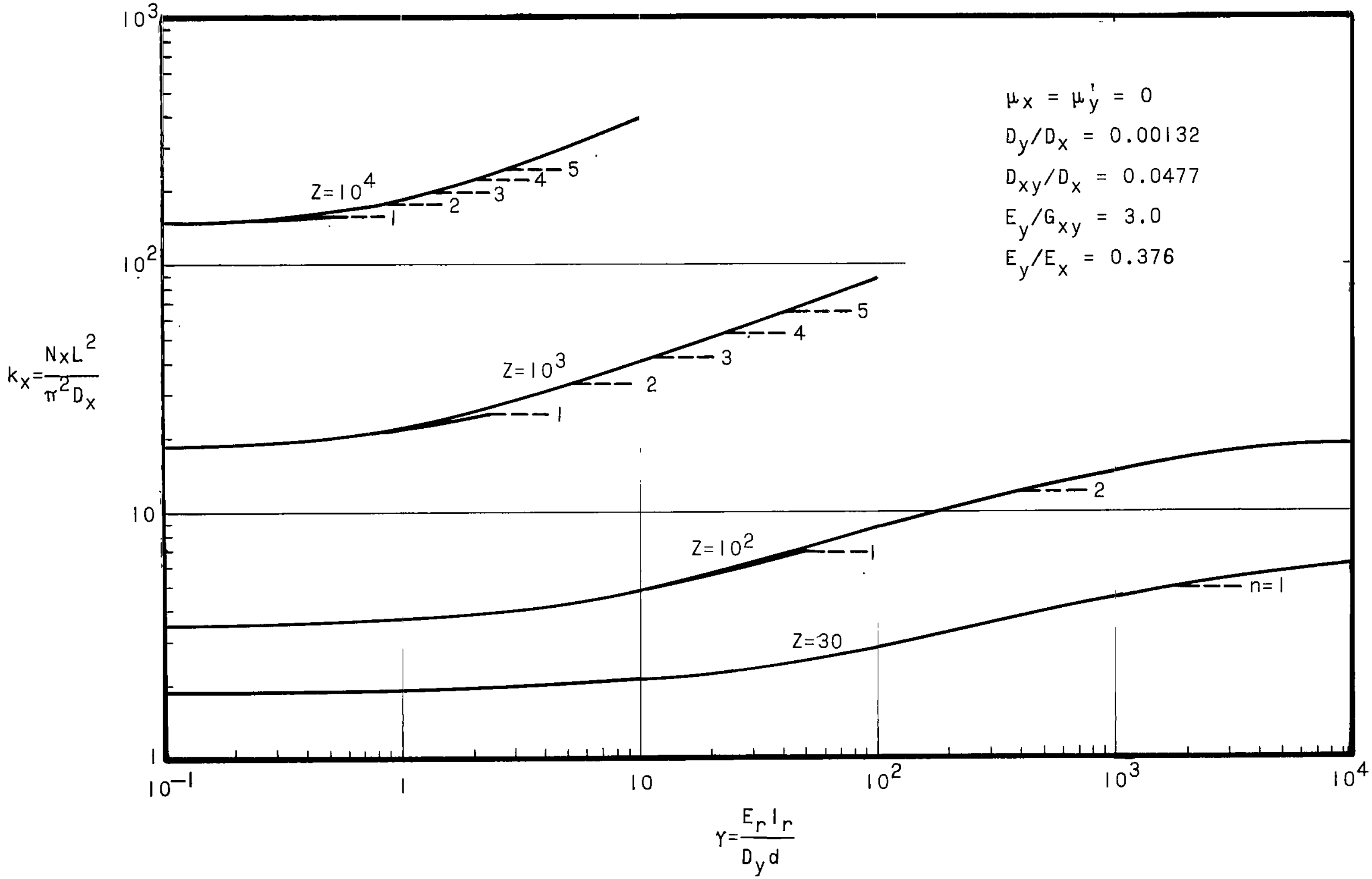


Figure 2.- Theoretical critical-stress coefficient of a ring-stiffened orthotropic cylinder in axial compression.

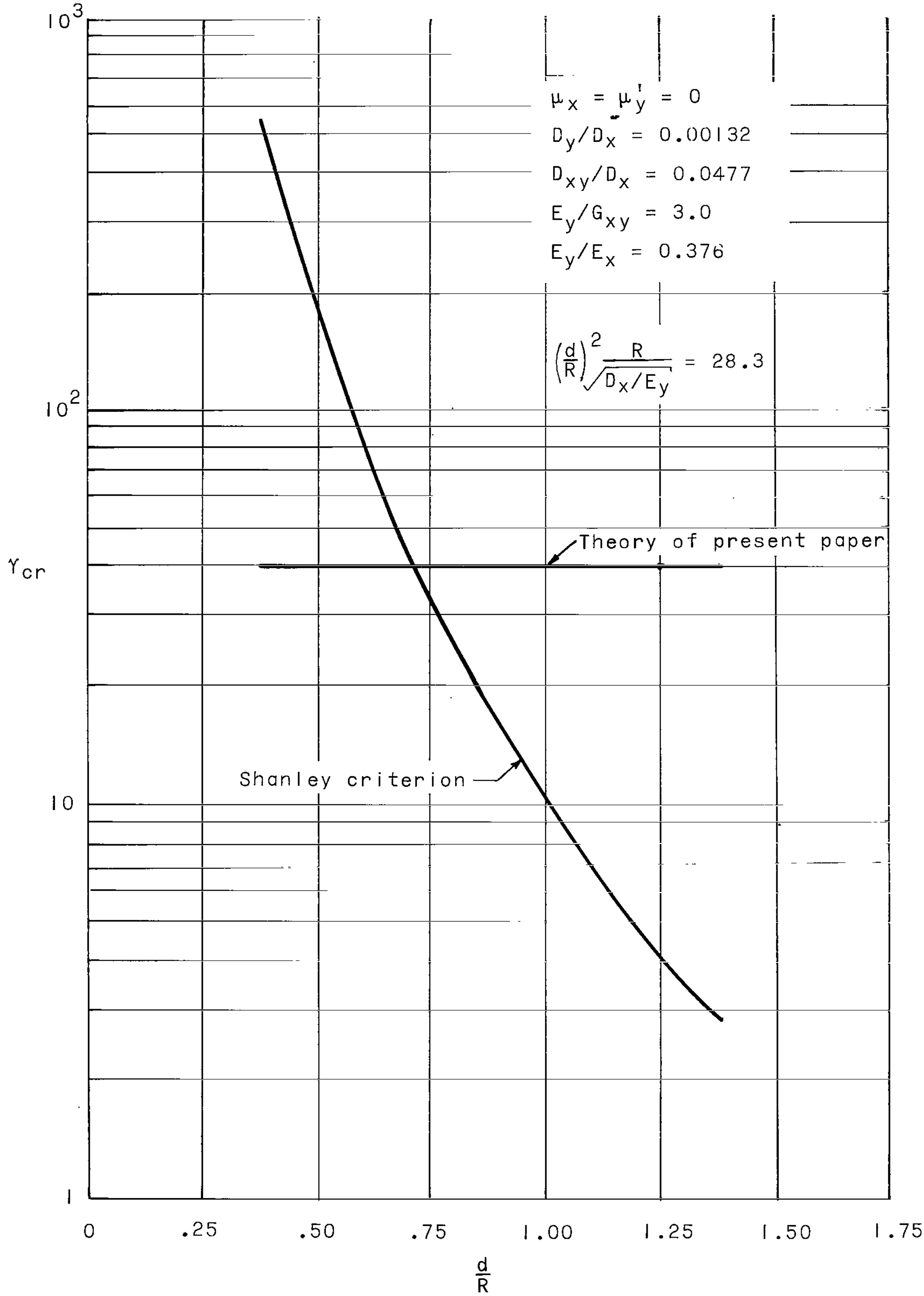


Figure 3.- Comparison of Shanley criterion with theory of present paper.

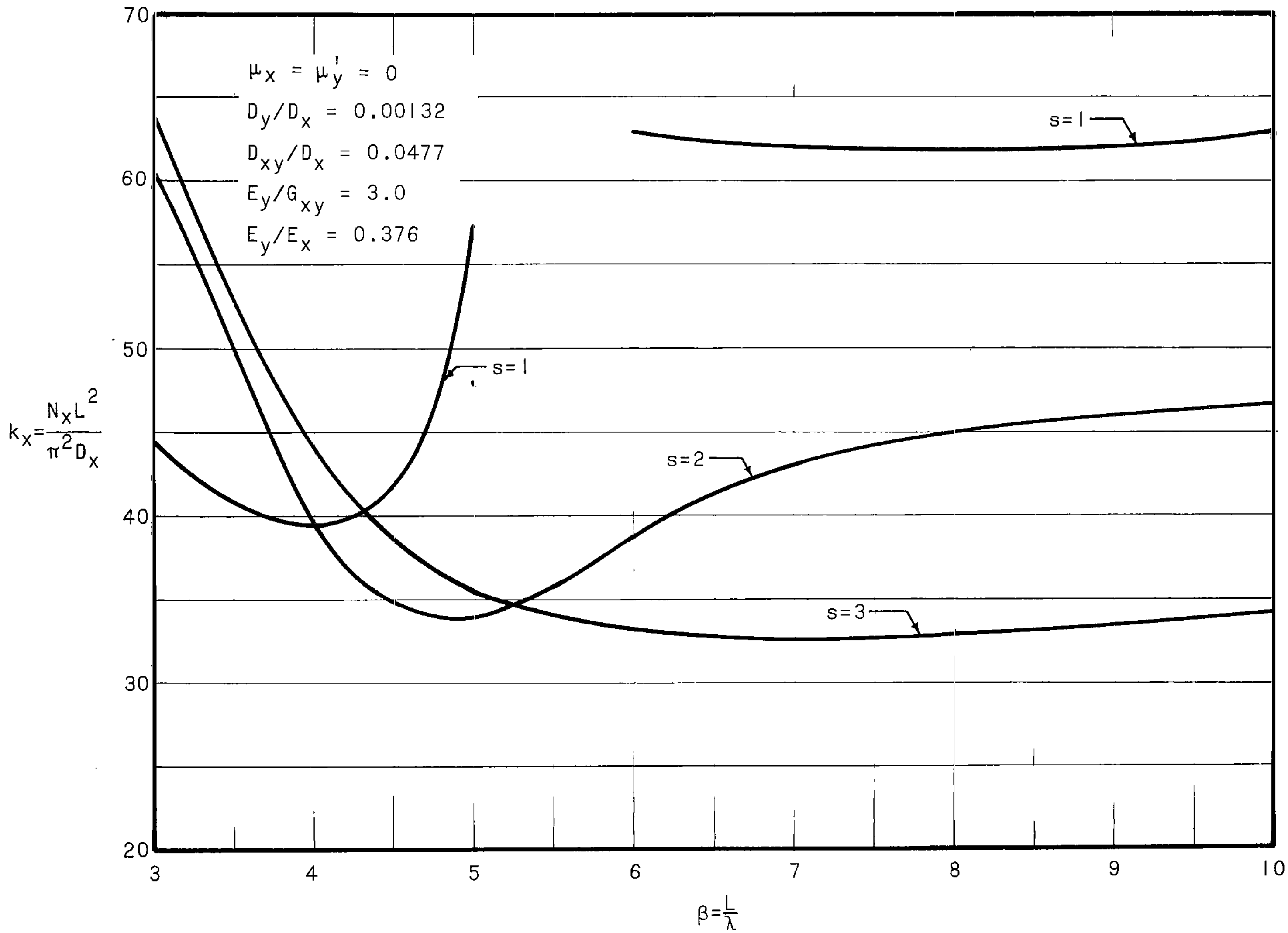
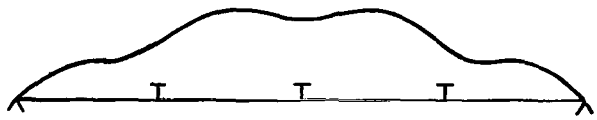


Figure 4.- Minimization of general instability equation for three rings. $Z = 452.8$; $\gamma = 62.5$.

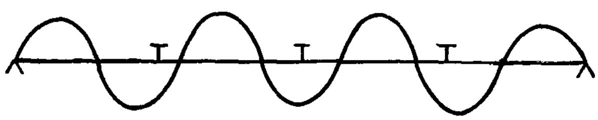
$k_x = 39.27,\ \beta = 4.1,\ s = 1$

$$w = +0.99386314 \sin\frac{\pi x}{L} + 0.10168799 \sin\frac{7\pi x}{L} - 0.04283381 \sin\frac{9\pi x}{L} + 0.00780309 \sin\frac{15\pi x}{L}$$



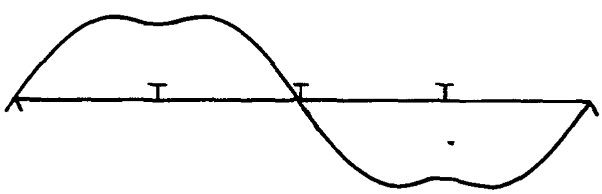
$k_x = 61.78,\ \beta = 8.0,\ s = 1$

$$w = +0.16775675 \sin\frac{\pi x}{L} + 0.98544122 \sin\frac{7\pi x}{L} - 0.02745249 \sin\frac{9\pi x}{L} + 0.00307095 \sin\frac{15\pi x}{L}$$



$k_x = 33.86,\ \beta = 4.8,\ s = 2$

$$w = +0.98884739 \sin\frac{2\pi x}{L} + 0.14648043 \sin\frac{6\pi x}{L} - 0.02547759 \sin\frac{10\pi x}{L} + 0.00852019 \sin\frac{14\pi x}{L}$$



$k_x = 32.66,\ \beta = 7.1,\ s = 3$

$$w = -0.74524561 \sin\frac{3\pi x}{L} - 0.66633409 \sin\frac{5\pi x}{L} + 0.02135362 \sin\frac{11\pi x}{L} - 0.01232324 \sin\frac{13\pi x}{L}$$

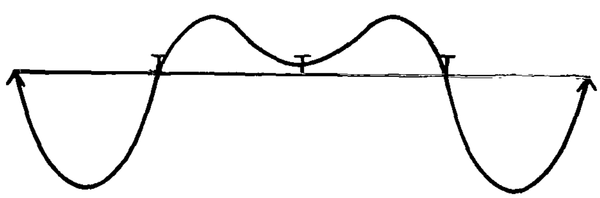


Figure 5.- Axial-mode shapes for minimum critical-stress coefficients of figure 4 (y held constant).

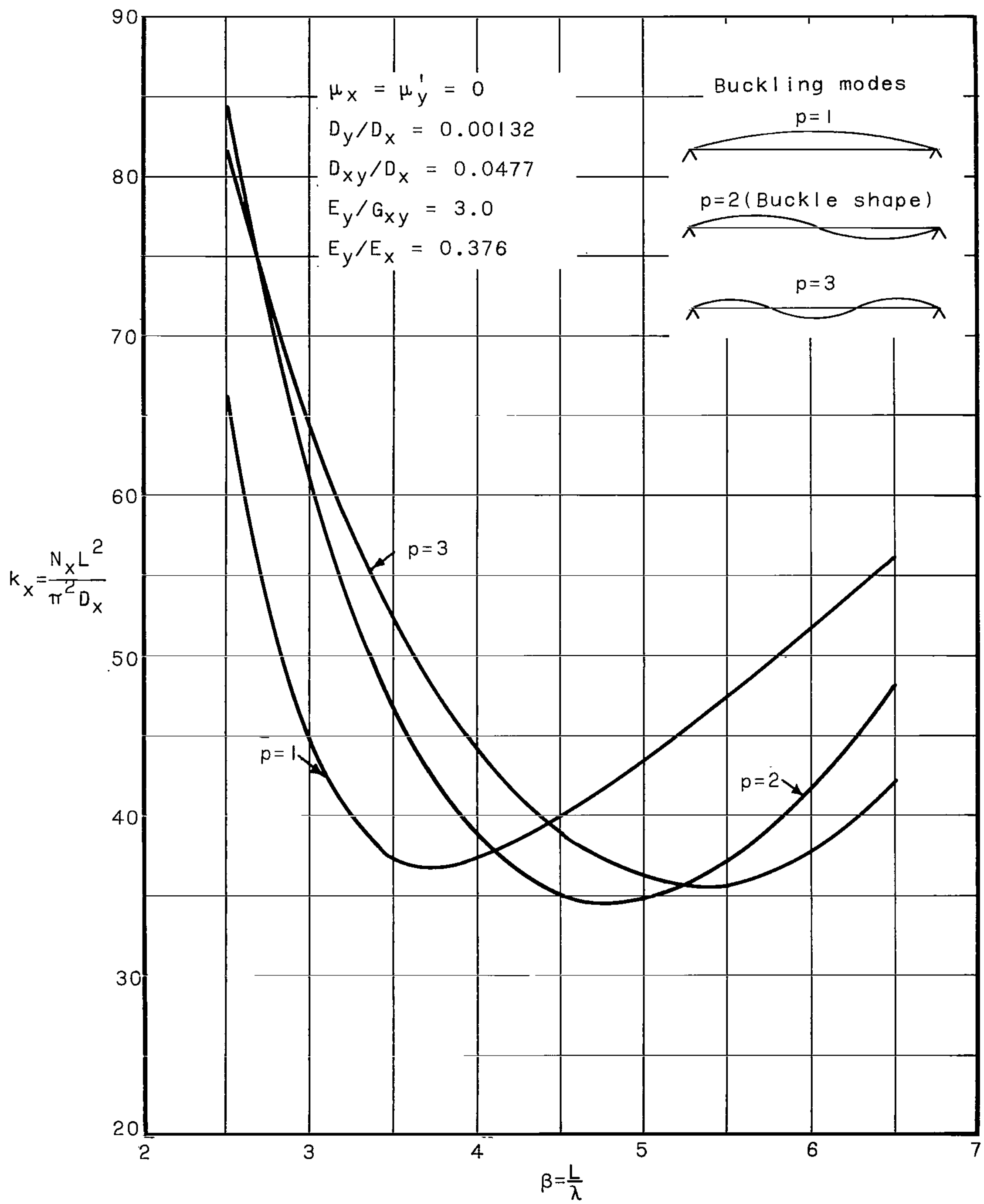


Figure 6.- Minimization of general instability equation and corresponding buckling modes for an infinite number of rings. $Z = 452.8$; $\gamma = 62.5$.

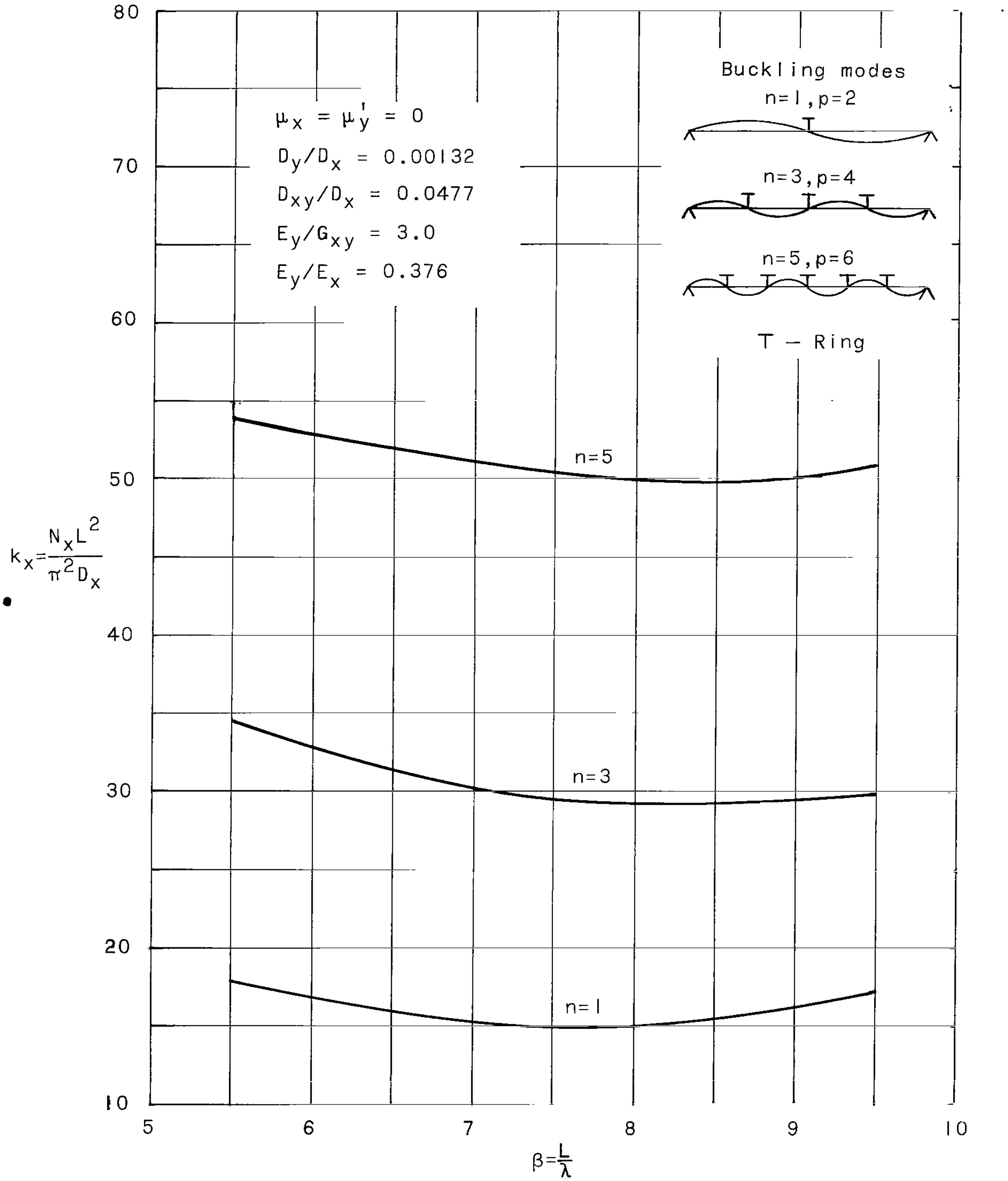


Figure 7.- Minimization of panel stability equation and corresponding buckling modes for one, three, and five rings. Z = 452.8.

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—National Aeronautics and Space Act of 1958

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